Fellow	Douglas **Svensson Seth**
Host Organisation	NTNU
Scientific coordinator	Mats **Ehrnström**

I – SCIENTIFIC ACTIVITY DURING YOUR FELLOWSHIP

The goal of my research programme at NTNU was to study water waves through the investigation of asymmetric solutions to the capillary-gravity Whitham equation, as well as considering internal waves in a three-dimensional setting with nonzero vorticity.

Asymmetric Solutions to the Capillary-Gravity Whitham Equation

The capillary-gravity Whitham equation is a model for shallow water waves with full dispersion. A previous paper by Ehrnström et al. provided a detailed description of all small even solutions to this equation. Due to the translational invariance of the equation, even solutions are the only possibility for small waves when they bifurcate from a point where the kernel of the linearization is one dimensional. However, with a two-dimensional kernel asymmetric waves this evenness is not guaranteed, and Asymmetric waves are a possibility.

Together with Ola Mæhlen (University of Oslo), I established the existence of certain symmetry breaking wave number pairs, which allowed us to find small asymmetric solutions using the surface tension strength as a parameter.

The results of this project are expected to be submitted to a journal as an article soon.

Internal Capillary-Gravity Waves in Three Dimensions with Vorticity

Building upon my previous work, I aimed to extend an existence result of capillary-gravity waves with vorticity to include internal waves on an arbitrary number of free interfaces. My previous paper had established the existence of doubly periodic travelling capillary-gravity waves over a Beltrami flow (proportional vorticity and velocity). In this project, I considered n free interfaces vertically sandwiched between n+1 Beltrami flows; each layer allowed to have a different proportionality factor between velocity and vorticity.

The mathematical model directly describes waves between immiscible fluids, such as oil and water, but can also serve as a model for internal waves in the ocean at a sharp change in density.

I was able to prove existence of nontrivial internal waves when all the proportionality factors are small. For larger vorticity, I identified an easy way to check if nontrivial solutions exist for a specific set of parameter values.

The results of this project have been submitted to a journal as an article.

Conclusion

This research project successfully achieved most of its objectives although not as quickly as we initially anticipated. Both projects can be followed up with additional research. The next step is most likely to determine if there exist asymmetric waves for the full water wave problem using the Euler equations.

II – PUBLICATION(S) DURING YOUR FELLOWSHIP

D. Svensson Seth, K. Varholm, E. Wahlén, *Symmetric doubly periodic gravity-capillary waves with small vorticity,* 2022 (Submitted)

D. Svensson Seth, *Symmetric doubly periodic gravity-capillary waves with small vorticity,* 2023 (Submitted)

O. Mæhlen, D. Svensson Seth, *Asymmetric travelling wave solutions of the capillary-gravity Whitham Equation (In preparation)*

III – ATTENDED SEMINARS, WORKHOPS, CONFERENCES

Seminars

DNA Seminar, NTNU, 2023 May 05
Title: Internal doubly periodic gravity-capillary waves with vorticity

Analysis and Differential Equations Seminar, Bath University, 2022 May 02
Title: Three-dimensional steady waves with vorticity: on the surface and internally

Joint mathematics seminar, Bergen University, 2022 March 22
Title: The three-dimensional steady water wave problem with vorticity

DNA Seminar, NTNU, 2021 November 22
Title: The three-dimensional water wave problem with vorticity

Workshops and Conferences

The Abel symposium 2023 June 13-16

Workshop on Analysis of PDEs, Karlsruhe Institute of Technology, 2023 March 27-31

Workshop on nonlinear dispersive equations, Wolfgang Pauli Institute, Vienna, 2022 Oktober 17-21
Talk: Asymmetric Solutions to the Capillary-Gravity Whitham Equation

2nd Norwegian meeting on PDEs, Bergen University, 2022 June 8-10

IV – RESEARCH EXCHANGE PROGRAMME (REP)

Host organization: INRIA Bordeaux (Cardamom research group)
Dates: 2023 May 8-12
Local coordinator: Martin Parisot

The visit allowed me to have interesting discussions with Martin Parisot, as well as some of the other Cardamom team members, gaining a deeper understanding of the modelling and numerical simulations of water waves.
I was also given the opportunity to give a presentation about my research to the Cardamom group.